Inference on Self-Exciting Jumps in Prices and Volatility using High Frequency Measures.

Supplementary Appendix: Results for models $\mathcal{M}_1$ to $\mathcal{M}_{11}$

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Abstract

In this supplement we provide additional posterior results that complement those documented in Section 4 of the main text. Specifically, we report Bayesian point and interval estimates of the static parameters of models $\mathcal{M}_1$ to $\mathcal{M}_{11}$ (specified in Table 2 of the main text). The prior distributions described in Table 5 (Appendix A of the main text) are employed - where appropriate - for the nested models. These prior distributions are also applied to the common parameters in the non-nested models $\mathcal{M}_5$ to $\mathcal{M}_7$, with the priors for the jump intensity parameters in those models being uniform and conforming to the theoretical restrictions that the model-implied unconditional jump intensities are between 0 and 1. The prior distributions employed for the realized GARCH specifications $\mathcal{M}_{10}$ to $\mathcal{M}_{11}$ conform to the stationarity conditions underpinning the model. All eleven models are estimated using the S&P500 data over the sample period from January 3, 1996 to June 23, 2014, inclusive. The marginal posterior means (MPMs), 95% highest posterior density (HPD) intervals, along with the inefficiency factors associated with the relevant MCMC draws are recorded in Tables A1 to A11, respectively. Each table also contains the model-implied instantaneous and time lagged co-jump statistics.

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Table A1: Posterior summaries for model $\mathcal{M}_1$, based on S&P500 stock index data from January 3, 1996 to June 23, 2014, inclusive.

$\mathcal{M}_1$: $\beta_{vp}^{(-)} = 0$

Parameter	MPM	95% HPD interval	Inefficiency Factor
μ	0.197	(0.137,0.253)	1.53
γ	-8.610	(-9.961,-5.540)	1.02
ρ	-0.355	(-0.420,-0.286)	7.35
μ_p	-0.412	(-0.438,-0.405)	6.70
γ_p	11.297	(10.081,12.390)	30.16
σ_p	0.208	(0.189,0.226)	14.37
π_p	0.382	(0.301,0.466)	11.43
α	$9.16e^{-4}$	($2.73e^{-5}$,$3.41e^{-3}$)	1.83
β	0.803	(0.613,0.948)	16.21
σ_{M_p}	0.182	(0.162,0.202)	15.34
ψ_0	1.044	(0.772,1.228)	204.06
ψ_1	1.303	(1.253,1.340)	144.57
σ_{BV}	0.436	(0.422,0.450)	6.59
κ	0.116	(0.092,0.142)	54.74
θ	$8.08e^{-3}$	($7.30e^{-3}$,$8.96e^{-3}$)	15.33
σ_v	0.016	(0.014,0.017)	20.22
μ_v	$9.25e^{-3}$	($7.80e^{-3}$,0.011)	34.11
δ_0^p	0.134	(0.109,0.173)	14.07
α_p	0.097	(0.073,0.128)	9.57
β_{pp}	0.062	(0.048,0.079)	11.67
δ_0^v	0.123	(0.084,0.165)	46.73
α_v	0.035	(0.024,0.047)	83.77
β_{vv}	0.031	(0.021,0.041)	86.48
β_{vp}	$5.94e^{-4}$	($1.43e^{-5}$,$2.04e^{-3}$)	1.75
$\Pr\left(\Delta N_t^v = 1 \vert \Delta N_t^p = 1\right)$	0.104	(0.063,0.151)	24.76
$\Pr\left(\Delta N_{t+1}^v = 1 \vert \Delta N_t^p = 1\right)$	0.114	(0.069,0.163)	27.59

Table A2: Posterior summaries for model $\mathcal{M}_2$, based on S&P500 stock index data from January 3, 1996 to June 23, 2014, inclusive.

$\mathcal{M}_2$: $\beta_{vp} = \beta_{vp}^{(-)} = 0$

Parameter	MPM	95% HPD interval	Inefficiency Factor
μ	0.196	(0.140,0.252)	1.52
γ	-8.674	(-9.963,-5.655)	0.98
ρ	-0.354	(-0.419,-0.290)	6.39
μ_p	-0.424	(-0.442,-0.407)	7.82
γ_p	11.655	(10.564,12.810)	25.73
σ_p	0.207	(0.189,0.224)	12.99
π_p	0.383	(0.301,0.462)	11.46
α	$8.57e^{-4}$	$(1.73e^{-5},3.28e^{-3})$	1.92
β	0.797	(0.625,0.945)	17.82
σ_{M_p}	0.183	(0.164,0.202)	12.72
ψ_0	1.114	(0.923,1.316)	130.50
ψ_1	1.316	(1.280,1.356)	94.00
σ_{BV}	0.436	(0.421,0.450)	6.60
κ	0.114	(0.091,0.137)	53.80
θ	$8.08e^{-3}$	$(7.31e^{-3},8.93e^{-3})$	13.79
σ_v	0.016	(0.014,0.017)	20.41
μ_v	$8.95e^{-3}$	$(7.55e^{-3},0.011)$	44.31
δ_0^p	0.135	(0.109,0.171)	15.03
α_p	0.097	(0.072,0.127)	10.63
β_{pp}	0.062	(0.047,0.080)	13.97
δ_0^v	0.122	(0.082,0.161)	43.19
α_v	0.032	(0.019,0.048)	170.82
β_{vv}	0.028	(0.017,0.042)	159.08
$\Pr\left(\Delta N_t^v = 1 \vert \Delta N_t^p = 1\right)$	0.108	(0.071,0.146)	16.05
$\Pr\left(\Delta N_{t+1}^v = 1 \vert \Delta N_t^p = 1\right)$	0.117	(0.075,0.157)	17.94

Table A3: Posterior summaries for model $\mathcal{M}_3$, based on S&P500 stock index data from January 3, 1996 to June 23, 2014, inclusive.

$\mathcal{M}_3$: $\Delta N_t^p = \Delta N_t^v$ for all $t = 1, ..., T$

Parameter	MPM	95% HPD interval	Inefficiency Factor
μ	0.232	(0.166,0.292)	1.14
γ	-7.910	(-9.925,-3.900)	1.11
ρ	-0.397	(-0.451,-0.340)	6.66
μ_p	-0.436	(-0.454,-0.419)	8.74
γ_p	12.178	(11.027,13.057)	36.97
σ_p	0.197	(0.177,0.217)	14.59
π_p	0.518	(0.243,0.795)	170.94
α	0.088	(0.081,0.096)	0.99
β	0.700	(0.406,0.923)	1.02
σ_{M_p}	0.195	(0.175,0.214)	14.02
ψ_0	1.245	(1.052,1.500)	185.76
ψ_1	1.357	(1.318,1.403)	129.77
σ_{BV}	0.455	(0.442,0.469)	3.51
κ	0.034	(0.026,0.043)	2.39
θ	0.014	(0.012,0.015)	1.17
σ_v	0.020	(0.018,0.021)	22.14
μ_v	$8.05e^{-3}$	($8.00e^{-3}$,$8.24e^{-3}$)	1.03
$\delta_0^p = \delta_0^v$	$2.22e^{-4}$	($5.41e^{-6}$,$8.35e^{-4}$)	1.03
$\alpha_p = \alpha_v$	0.140	(0.044,0.282)	20.97
$\beta_{pp} = \beta_{vv}$	0.078	($4.24e^{-3}$,0.203)	10.25
$\Pr\left(\Delta N_t^v = 1 \mid \Delta N_t^p = 1\right)$	1.00	N/A	N/A
$\Pr\left(\Delta N_{t+1}^v = 1 \mid \Delta N_t^p = 1\right)$	0.00	N/A	N/A

Table A4: Posterior summaries for model $\mathcal{M}_4$, based on S&P500 stock index data from January 3, 1996 to June 23, 2014, inclusive.

$\mathcal{M}_4$: $\Delta N_t^v = 0$ for all $t = 1, ..., T$

Parameter	MPM	95% HPD interval	Inefficiency Factor
μ	0.205	(0.142,0.263)	1.40
γ	-8.270	(-9.946,-4.630)	1.12
ρ	-0.336	(-0.388,-0.283)	5.15
μ_p	-0.436	(-0.454,-0.419)	6.90
γ_p	12.728	(11.617,13.921)	22.18
σ_p	0.210	(0.191,0.227)	12.42
π_p	0.405	(0.330,0.484)	11.91
α	$8.51e^{-4}$	$(2.19e^{-5}, 3.20e^{-3})$	1.82
β	0.800	(0.602,0.941)	22.64
σ_{M_p}	0.182	(0.162,0.201)	13.54
ψ_0	1.340	(1.157,1.518)	130.98
ψ_1	1.366	(1.330,1.402)	92.92
σ_{BV}	0.450	(0.437,0.464)	3.85
κ	0.036	(0.028,0.045)	2.43
θ	0.013	(0.012,0.015)	1.11
σ_v	0.020	(0.018,0.021)	18.91
δ_0^p	0.138	(0.110,0.177)	17.63
α_p	0.097	(0.072,0.127)	11.07
β_{pp}	0.062	(0.047,0.078)	14.67
$\Pr\left(\Delta N_t^v = 1 \mid \Delta N_t^p = 1\right)$	0.00	N/A	N/A
$\Pr\left(\Delta N_{t+1}^v = 1 \mid \Delta N_t^p = 1\right)$	0.00	N/A	N/A

Table A5: Posterior summaries for model $\mathcal{M}_5$, based on S&P500 stock index data from January 3, 1996 to June 23, 2014, inclusive.

$$\mathcal{M}_5\text{: } \begin{cases} \delta_t^p = \alpha_{p_0} + \alpha_p V_t \text{ and} \\ \delta_t^v = \alpha_{v_0} + \alpha_v V_t \text{ for all } t = 1, ..., T \end{cases}$$

Parameter	MPM	95% HPD interval	Inefficiency Factor
μ	0.196	(0.135,0.252)	1.54
γ	-8.639	(-9.960,-5.533)	1.04
ρ	-0.328	(-0.390,-0.264)	6.31
μ_p	-0.420	(-0.437,-0.403)	7.40
γ_p	11.223	(10.055,12.249)	26.18
σ_p	0.208	(0.189,0.226)	13.85
π_p	0.381	(0.291,0.467)	16.51
α	$9.24e^{-4}$	($2.45e^{-5}$,$3.44e^{-3}$)	1.64
β	0.770	(0.563,0.941)	23.25
σ_{M_p}	0.183	(0.163,0.203)	14.41
ψ_0	1.062	(0.865,1.248)	152.61
ψ_1	1.308	(1.270,1.346)	111.28
σ_{BV}	0.440	(0.426,0.453)	5.07
κ	0.087	(0.073,0.100)	17.21
θ	$9.13e^{-3}$	($8.20e^{-3}$,0.010)	11.38
σ_v	0.016	(0.015,0.018)	23.18
μ_v	0.012	($9.63e^{-3}$,0.015)	51.06
δ_0^p	0.140	(0.111,0.189)	21.36
α_p	0.161	($3.73e^{-3}$,0.597)	2.02
δ_0^v	0.066	(0.046,0.086)	22.23
α_v	2.807	(2.338,2.967)	33.32
$\Pr\left(\Delta N_t^v = 1 \mid \Delta N_t^p = 1\right)$	0.052	(0.032,0.075)	8.01
$\Pr\left(\Delta N_{t+1}^v = 1 \mid \Delta N_t^p = 1\right)$	0.056	(0.035,0.080)	8.90

Table A6: Posterior summaries for model $\mathcal{M}_6$, based on S&P500 stock index data from January 3, 1996 to June 23, 2014, inclusive.

$$\mathcal{M}_6\text{:} \begin{cases} \delta_t^p = \alpha_{p_0} + \alpha_{p1} V_t + \alpha_{p2} V_t^2 \text{ and} \\ \delta_t^v = \alpha_{v_0} + \alpha_{v1} V_t + \alpha_{v2} V_t^2 \text{ for all } t = 1, ..., T \end{cases}$$

Parameter	MPM	95% HPD interval	Inefficiency Factor
μ	0.192	(0.1304,0.250)	1.57
γ	-8.589	(-9.959,-5.460)	0.99
ρ	-0.324	(-0.389,-0.259)	7.67
μ_p	-0.419	(-0.436,-0.403)	7.68
γ_p	11.103	(10.060,12.391)	32.74
σ_p	0.209	(0.187,0.226)	16.86
π_p	0.383	(0.301,0.464)	12.13
α	$9.53e^{-4}$	($2.60e^{-5}$,$3.46e^{-3}$)	1.88
β	0.701	(0.431,0.923)	40.62
σ_{M_p}	0.183	(0.162,0.205)	17.80
ψ_0	1.037	(0.863,1.253)	144.04
ψ_1	1.303	(1.268,1.344)	103.62
σ_{BV}	0.441	(0.428,0.455)	5.00
κ	0.081	(0.069,0.095)	17.70
θ	$9.52e^{-3}$	($8.55e^{-3}$,0.011)	9.62
σ_v	0.017	(0.015,0.018)	20.60
μ_v	0.013	(0.010,0.016)	36.47
δ_0^p	0.158	(0.112,0.247)	18.45
α_{p1}	0.425	($9.07e^{-3}$,1.443)	6.12
α_{p2}	-1.977	(-9.317,5.539)	1.21
δ_0^v	0.054	(0.038,0.073)	18.45
α_{v1}	2.549	(1.547,2.950)	10.56
α_{v2}	0.790	(-6.394,8.867)	1.34
$\Pr\left(\Delta N_t^v = 1 \vert \Delta N_t^p = 1\right)$	0.046	(0.022,0.076)	24.54
$\Pr\left(\Delta N_{t+1}^v = 1 \vert \Delta N_t^p = 1\right)$	0.049	(0.024,0.078)	23.05

Table A7: Posterior summaries for model $\mathcal{M}_7$, based on S&P500 stock index data from January 3, 1996 to June 23, 2014, inclusive.

$$\mathcal{M}_7: \begin{cases} \delta_t^p = \frac{\exp\left(\alpha_{p_0}+\alpha_p V_t\right)}{1+\exp\left(\alpha_{p_0}+\alpha_p V_t\right)} \text{ and} \\ \delta_t^v = \frac{\exp\left(\alpha_{v_0}+\alpha_v V_t\right)}{1+\exp\left(\alpha_{v_0}+\alpha_v V_t\right)} \text{ for all } t=1,...,T \end{cases}$$

Parameter	MPM	95% HPD interval	Inefficiency Factor
μ	0.202	(0.143,0.258)	1.58
γ	-8.759	(-9.963,-5.777)	1.06
ρ	-0.322	(-0.389,-0.254)	6.41
μ_p	-0.419	(-0.435,-0.402)	7.91
γ_p	11.134	(9.958,12.241)	33.88
σ_p	0.209	(0.191,0.226)	13.13
π_p	0.385	(0.307,0.471)	11.48
α	$9.02e^{-4}$	($2.43e^{-5}$,$3.20e^{-3}$)	1.76
β	0.759	(0.534,0.944)	36.90
σ_{M_p}	0.182	(0.162,0.201)	13.95
ψ_0	1.024	(0.827,1.230)	210.16
ψ_1	1.298	(1.258,1.338)	151.42
σ_{BV}	0.429	(0.415,0.443)	6.36
κ	0.143	(0.110,0.161)	51.51
θ	$8.04e^{-3}$	($7.31e^{-3}$,$8.87e^{-3}$)	15.21
σ_v	0.016	(0.015,0.018)	22.10
μ_v	$8.31e^{-3}$	($7.05e^{-3}$,0.010)	47.81
δ_0^p	0.143	(0.110,0.200)	43.62
α_{p_0}	-1.823	(-2.108,-1.417)	40.32
α_p	1.171	(0.035,3.877)	2.95
δ_0^v	0.159	(0.111,0.209)	75.80
α_{v_0}	-3.083	(-3.430,-2.771)	34.50
α_v	67.697	(51.550,86.089)	117.67
$\Pr\left(\Delta N_t^v = 1 \mid \Delta N_t^p = 1\right)$	0.139	(0.086,0.194)	36.05
$\Pr\left(\Delta N_{t+1}^v = 1 \mid \Delta N_t^p = 1\right)$	0.147	(0.091,0.205)	37.04

Table A8: Posterior summaries for model $\mathcal{M}_8$, based on S&P500 stock index data from January 3, 1996 to June 23, 2014, inclusive.

$\mathcal{M}_8$: $\delta_t^p = \delta_0^p$, $\delta_t^v = \delta_0^v$ for all $t = 1, ..., T$

Parameter	MPM	95% HPD interval	Inefficiency Factor
μ	0.195	(0.135,0.250)	1.59
γ	-8.543	(-9.960,-5.331)	1.09
ρ	-0.331	(-0.392,-0.270)	6.86
μ_p	-0.422	(-0.442,-0.405)	11.28
γ_p	11.397	(10.135,12.977)	49.19
σ_p	0.210	(0.191,0.227)	13.29
π_p	0.385	(0.304,0.469)	13.29
α	$8.94e^{-4}$	($2.28e^{-5}$,$3.28e^{-3}$)	1.82
β	0.764	(0.540,0.943)	24.77
σ_{M_p}	0.182	(0.162,0.203)	15.01
ψ_0	1.101	(0.852,1.374)	283.17
ψ_1	1.318	(1.269,1.368)	204.54
σ_{BV}	0.447	(0.433,0.460)	4.57
κ	0.052	(0.042,0.063)	14.12
θ	0.011	($9.36e^{-3}$,0.012)	6.51
σ_v	0.017	(0.015,0.018)	30.89
μ_v	0.017	(0.012,0.023)	86.26
δ_0^p	0.142	(0.110,0.197)	23.80
δ_0^v	0.024	(0.015,0.036)	18.89
$\Pr\left(\Delta N_t^v = 1 \vert \Delta N_t^p = 1\right)$	0.018	(0.007,0.030)	7.61
$\Pr\left(\Delta N_{t+1}^v = 1 \vert \Delta N_t^p = 1\right)$	0.019	(0.007,0.033)	10.84

Table A9: Posterior summaries for model $\mathcal{M}_9$, based on S&P500 stock index data from January 3, 1996 to June 23, 2014, inclusive.

$\mathcal{M}_9$: $\delta_t^p = 0$, $\delta_t^v = 0$ for all $t = 1, ..., T$

Parameter	MPM	95% HPD interval	Inefficiency Factor
μ	0.217	(0.159,0.272)	1.18
γ	-7.916	(-9.916,-3.922)	1.21
ρ	-0.287	(-0.342,-0.229)	4.90
ψ_0	0.309	(0.222,0.404)	19.02
ψ_1	1.090	(1.071,1.110)	23.31
σ_{BV}	0.470	(0.457,0.483)	19.60
κ	0.038	(0.029,0.050)	1.52
θ	0.013	(0.012,0.015)	1.02
σ_v	0.023	(0.021,0.025)	11.43
$\Pr\left(\Delta N_t^v = 1 \vert \Delta N_t^p = 1\right)$	0.000	N/A	N/A
$\Pr\left(\Delta N_{t+1}^v = 1 \vert \Delta N_t^p = 1\right)$	0.000	N/A	N/A

Table A10: Posterior summaries for model $\mathcal{M}_{10}$, based on S&P500 stock index data from January 3, 1996 to June 23, 2014, inclusive.

$$\mathcal{M}_{10}: \begin{cases} r_t = \sqrt{h_t} z_t \\ h_t = \omega + \beta h_{t-1} + \gamma BV_{t-1} \text{ and} \\ BV_t = \xi + \varphi h_t + \tau_1 z_t + \tau_2 \left(z_t^2 - 1\right) + u_t \end{cases}$$

Parameter	MPM	95% HPD interval	Inefficiency Factor
ω	0.008	(0.007,0.009)	346.80
β	0.615	(0.570,0.647)	403.06
γ	0.083	(0.067,0.093)	494.23
ξ	-0.079	(-0.103,-0.068)	165.17
φ	3.901	(3.454,4.615)	161.66
τ_1	-0.002	(-0.003,-0.001)	3.38
τ_2	0.003	(0.003,0.004)	3.47
σ_u	0.034	(0.033,0.034)	3.42
$\Pr\left(\Delta N_t^v = 1 \mid \Delta N_t^p = 1\right)$	0.000	N/A	N/A
$\Pr\left(\Delta N_{t+1}^v = 1 \mid \Delta N_t^p = 1\right)$	0.000	N/A	N/A

Table A11: Posterior summaries for model $\mathcal{M}_{11}$, based on S&P500 stock index data from January 3, 1996 to June 23, 2014, inclusive.

$$\mathcal{M}_{11}: \begin{cases} r_t = \sqrt{h_t} z_t \\ \ln h_t = \omega + \beta \ln h_{t-1} + \gamma \ln BV_{t-1} \text{ and} \\ \ln BV_t = \xi + \varphi \ln h_t + \tau_1 z_t + \tau_2 \left(z_t^2 - 1\right) + u_t \end{cases}$$

Parameter	MPM	95% HPD interval	Inefficiency Factor
ω	-0.501	(-0.587,0.427)	23.48
β	0.527	(0.492,0.555)	94.80
γ	0.363	(0.336,0.386)	78.55
ξ	0.742	(0.525,0.928)	11.45
φ	1.173	(1.125,1.214)	11.72
τ_1	-0.094	(-0.106,-0.084)	3.41
τ_2	0.059	(0.054,0.063)	3.46
σ_u	0.524	(0.513,0.533)	3.38
$\Pr\left(\Delta N_t^v = 1 \mid \Delta N_t^p = 1\right)$	0.000	N/A	N/A
$\Pr\left(\Delta N_{t+1}^v = 1 \mid \Delta N_t^p = 1\right)$	0.000	N/A	N/A